COLORS FOR LEVEL PLOTS

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January 11, 1985.

ABSTRACT

The Friele-MacAdam-Chickering FMC-1 formula appears to provide a good automatic spectrum of equally spaced colors for use in color level plots. Other candidates include: the CIE *Luv* uniform color space, the Optical Society of America Ljg space, the MacLeod—Boynton space, and the Alvy Ray Smith hexcone model.

To get a C routine generating the FMC rainbow, type

```
mail netlib@research.att.com
send rainbow.c from graphics
```


1. Introduction

As hardware costs drop, color level plots are becoming a popular way to display scalar functions of two variables. Unlike surface plots with hidden lines removed, no part of the surface is obscured, and unlike ordinary contour plots, the relative heights of two separated peaks can be compared at a glance. But in current practice, the user must choose the color spectrum by hand to achieve good results. While a certain amount of artistic judgement will always be needed in the strategic choice of assigning, say, red to high and blue to low values, it ought to be possible to automate the tedious tuning of intermediate colors.

How can colors be chosen that are perceived by the human eye as separated by equal steps? One answer is to use numerical optimization to choose equally spaced points according to metrics from the color vision literature.

2. Color theory

This section summarizes the color theory needed for our purposes. See [1, 2, 3] for more details.

Any color can be characterized by three standard chromaticity coordinates *X*, *Y*, and *Z*. Given the spectral energy distribution *s* of a colored light source, form inner products

$$X = \langle s, \overline{x} \rangle$$

$$Y = \langle s, \overline{y} \rangle$$

$$Z = \langle s, \overline{z} \rangle .$$

with the three distributions given in Figure 1. Often it is more convenient to work with coordinates *x*, *y*, and *z* scaled so that $x + y + z = 1$. It is an experimental fact that two patches having the same x, y coordinates will be perceived as identical colors. The pure monochromatic colors lie on the surfboard curve in Figure 2, magentas line on a straight line between red and blue, and average daylight (point C) lies in the middle. Hue corresponds to angle about C; saturation corresponds to distance from C to the monochrome curve. The colors that can be physically realized on a CRT lie in a triangle; see Figures 3 and 4. Euclidean distance $\sqrt{x^2 + y^2}$ is *not* the difference metric our eyes use. The ellipses in Figure 2 illustrate that a comparatively large change near green is needed to make a change in hue that appears as large as a comparatively small change near blue.

There are many color coordinate systems and metrics in use. For some metrics, computation of equally spaced points is easy; for others, the metric forces more elaborate computations but could be closer to perceived color changes. Think of cartographic projections: no single mapping from the surface of a sphere to a plane will always be adequate (and a geoid may be better than a sphere), so in exploring equally spaced colors it is best to allow an arbitrary metric.

Display devices expect colors to be defined not by X,Y,Z but, essentially, by three values R,G,B in the interval $[0,1]$. For given phosphor chromaticity coordinates, red (x_1,y_1), green (x_2,y_2), blue (x_3,y_3), and white point (x_0,y_0), the transformation matrix is derived as follows [4]. Define $z=1-x-y$. Solve the linear system

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x_0/y_0 \\ 1 \\ z_0/y_0 \end{bmatrix}$$

for a_1, a_2, a_3. (In other words, let a_1, a_2, a_3 be the barycentric coordinates of white.) Then the transformation matrix from R,G,B to X,Y,Z is

$$\begin{bmatrix} a_1x_1 & a_2x_2 & a_3x_3 \\ a_1y_1 & a_2y_2 & a_3y_3 \\ a_1z_1 & a_2z_2 & a_3z_3 \end{bmatrix}.$$

For our **Hitachi** monitor, model HM-2719C-11, the phosphor chromaticities are [5]

	x	y
red	.60	.33
green	.22	.63
blue	.15	.064

Photometric measurements give relative luminances of $r=.1491$, $g=.6396$, $b=.2112$. Using the D65 white point, $x=.3217$, $y=.3290$, yields the transformations

$$\begin{bmatrix} .6074 & .2084 & .1620 \\ .3341 & .5968 & .0691 \\ .0709 & .1421 & .8487 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} 2.0317 & -.6294 & -.3365 \\ -1.1398 & 2.0618 & .0496 \\ .0212 & -.2926 & 1.1980 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

The monitor is driven by a Raster Technologies frame buffer; to convert from R,G,B to the color indices used in the frame buffer, use $r=\mathrm{floor}[255.999R^{1/\gamma}]$ and similarly for green and blue. Photometric measurements suggest $\gamma=2$ is better than, say, $\gamma=2.3$, but further calibration would be desirable. For our **Barco** monitor, with EBU phosphors, the chromaticities are [6]

	x	y
red	.64	.33
green	.29	.60
blue	.15	.06

Using the D65 white point yields

$$\begin{bmatrix} .4722 & .3321 & .1735 \\ .2435 & .6871 & .0694 \\ .0221 & .1260 & .9136 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} 2.7930 & -1.2704 & -.4338 \\ -.9968 & 1.9293 & .0427 \\ .0698 & -.2352 & 1.0992 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

For the **Lisp machine** Amtron color monitor, the chromaticities are

	x	y
red	.63	.35
green	.22	.64
blue	.15	.06

Using the D65 white point yields

$$\begin{bmatrix} .5991 & .2055 & .1731 \\ .3329 & .5979 & .0693 \\ .0190 & .1308 & .9119 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} 2.0256 & -.6225 & -.3373 \\ -1.1418 & 2.0517 & .0610 \\ .1215 & -.2813 & 1.0949 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

There is a projective transformation of x, y space, known as the CIE *Luv* uniform color space, defined by

$$u' = \frac{2x}{6y - x + 1.5}$$
$$v' = \frac{4.5y}{6y - x + 1.5}.$$

See Figure 5. (There is also an older version of this formula using $3y$ instead of $4.5y$ in the numerator of v; the experiments here confirm that the newer system provides better spacing.) Let $R = G = B = 1$ be taken as a reference white, with corresponding coordinates u'_0, v'_0. For two colors $i = 1, 2$ define

$$L_i = 116 Y_i^{1/3} - 16,$$
$$u_i^{star} = 13 L_i (u'_i - u'_0),$$
$$v_i^{star} = 13 L_i (v'_i - v'_0).$$

Then the color difference may be estimated by $\sqrt{(\Delta L)^2 + (\Delta u^{star})^2 + (\Delta v^{star})^2}$.

Another color space has been proposed by MacLeod and Boynton [7]. It can be applied as in the *Luv* case substituting r and b for u and v, where:

$$R_c = .15514x + .54312y - .03286z$$
$$G_c = -.15514x + .45684y + .03286z$$
$$B_c = .01608z$$
$$r = \frac{R_c}{R_c + G_c}$$
$$b = \frac{B_c}{R_c + G_c}$$

R_c represents the response of the red cones of the retina, and so on. MacLeod and Boynton do not encourage use of this space for color difference calculations and do not suggest how to factor in luminance differences; this has just been included in the tests as one more reasonable alternative. If one's aim is to make the separation between regions most visible, Frome's [8] findings indicate that only the difference between R and G_c is important (if the two regions have equal luminance); B_c makes no contribution to the subjective distinctness of borderlines. So a variant was derived by dropping b from the color difference formula.

Another important measure of color difference is defined by the Optical Society of America [9,p.165]. Using R, G, B defined by

$$\begin{bmatrix} .799 & .4194 & -.1648 \\ -.4493 & 1.3265 & .0927 \\ -.1149 & .3394 & .717 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

let

$$
\begin{aligned}
Y_0 &= 100Y(4.4934x^2 + 4.3034y^2 - 4.276xy - 1.3744x - 2.56439y + 1.8103) \\
L &= 5.9[Y_0^{1/3} - 2/3 + .042(Y_0 - 30)^{1/3}] \\
C &= L/[5.9(Y_0^{1/3} - 2/3)] \\
j &= C(1.7R^{1/3} + 8G^{1/3} - 9.7B^{1/3}) \\
g &= C(-13.7R^{1/3} + 17.7G^{1/3} - 4B^{1/3}).
\end{aligned}
$$

The color difference is then defined by $\sqrt{(\Delta L)^2 + (\Delta j)^2 + (\Delta g)^2}$.

The Friele-MacAdam-Chickering FMC-1 is a nonlinear formula for color differences [10] based more directly than the other formulas on the measured ellipses of equal color difference displayed in Figure 2. Define a P, Q, S coordinate system by

$$
\begin{aligned}
P &= .724X + .382Y - .098Z \\
Q &= -.48X + 1.37Y + .1276Z \\
S &= .686Z.
\end{aligned}
$$

Given two colors, compute averages and differences according to $P = (P_1 + P_2)/2$, $\Delta P = P_1 - P_2$ and similarly for Q, S, Y. Define intermediate quantities

$$
\begin{aligned}
D &= \sqrt{P^2 + Q^2} \\
a &= \sqrt{17.3e-6D^2 \ / \ (1 + 2.73P^2Q^2/(P^4 + Q^4))} \\
b &= \sqrt{3.098e-4(S^2 + .2015Y^2)} \\
\Delta C_3 &= (Q\Delta P - P\Delta Q)/(aD). \\
\Delta C_1 &= S(P\Delta P + Q\Delta Q)/(bD^2) - \Delta S/b \\
\Delta L &= .279(P\Delta P + Q\Delta Q)/(aD)
\end{aligned}
$$

Then the color difference is $\sqrt{(\Delta L)^2 + (\Delta C_1)^2 + (\Delta C_3)^2}$.

An even more sophisticated difference metric would use this MacAdam metric locally and compute geodesics, as is done by Muth and Persels [11, 12]. I have not pursued this possibility, although it fits into the same framework. Also, I ignore surround effects and Mach banding. I am interested in reproducing colors faithfully in 35mm transparencies and other media, but that is a separate problem not discussed here.

These formulas by no means exhaust the possibilities considered in the color literature, but are a representative sample. MacAdam [13, p.129] notes that "the search for [a uniform chromaticity diagram] has extended over 50 years and seems no nearer its goal than at the beginning."

3. Choosing a spacing of colors

An effective ordering of colors is given by the physical spectrum. One could conceivably use some other order, or add less saturated colors, but the spectrum provides a standard, easily described sequence. For plotting periodic data, use the physical spectrum augmented by the line of magentas. Color monitors can not generate a true violet, and magenta looks like a color intermediate between blue and red, so there is a danger of confusion if magenta is used as an approximate violet. White and black are left for labels, grid lines, and regions where the function is undefined.

One easy way to generate a spectrum is to linearly interpolate in R,G,B space between $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. A second, very similar spectrum is given by equally spaced values in the hexcone model of Alvy Ray Smith [14], which linearly interpolates in R,G,B space between $(1,0,0)$, $(1,1,0)$, $(0,1,0)$, $(0,1,1)$, and $(0,0,1)$.

These two techniques are very simple and are widely used in computer graphics but suffer from nonuniform spacing of colors. For example, the distance from pure red to its neighbor appears much larger than the distance from pure green to its neighbors. It is possible to do better by using numerical optimization to vary the hue points, starting from the hexcone spectrum and using the color theory metric described in the previous section. Taking h in the hexcone model as the optimization parameter forces the colors stay on the boundary of the physically realizable region. To eliminate the constraints that the hues be monotonic, change variables according to

$$\theta_j = \log \frac{h_{j+1} - h_j}{h_{j+2} - h_{j+1}},$$

for $1 \le j \le n-2$, where the boundary hues $h_1 = 0$ (red) and $h_n = 1$ (blue) are fixed. Thus θ_j should minimize

$$\sum_{i=1}^{n-1} (d(h_j, h_{j+1}) - \bar{d})^2$$

where d is the color difference metric and $\bar{d}$ is the average of the current color differences.

One possibility is to treat this as a general nonlinear least squares problem (with $n-2$ variables) and call the routine N2F by Gay from the PORT library or the earlier version NL2SOL [15]. A potential difficulty is that the piecewise linear boundary of the color gamut introduces jump discontinuities in the derivative of the optimization criterion, but in practice the method has been successful. Another approach to the optimization would be to treat it as a sequence of univariate zero-finding problems: given an estimated $d^{star} \approx d(h_{n-1}, h_n)$, choose θ_1 (and therefore h_2) so that $d(h_{1,} h_2) = d^{star}$, then choose θ_2 so that $d(h_{2,} h_3) = d^{star}$, and so on, finally getting a residual $d^{star} - d(h_{n-1}, h_n)$ which is driven to zero in an outer loop. The routine ZEROIN by Brent [16] would be suitable for this purpose.

In the special case of Luv if we are willing to ignore the L component of color difference, we can compute equally spaced colors without solving a system on nonlinear equations. Note that equally spaced points in u', v' can be mapped back to x, y by

$$x = (3/2) \frac{u}{u - (8/3)v + 2}$$

$$y = (2/3) \frac{v}{u - (8/3)v + 2}$$

A first guess at equally spaced colors along the boundary of the triangle of realizable colors in u, v space can be obtained by equally spaced steps in arc length along the boundary. This is not quite right because an interval spanning a vertex, for example green, overestimates the distance between the colors at the endpoints of that interval. See Figure 6. If d is the distance between colors and σ is the arclength distance from red to blue $|r-g| + |g-b|$ the correct d is somewhat smaller than σ/n; the same number of colors are assigned to both edges in the exact version as in the first guess. Say color c_k is on the blue to green edge and c_{k+1} is on the green to red edge. If the coordinates of the red, green, and blue vertices are w_1, w_2, and w_3 respectively, then

$$c_k = w_3 + dk(w_2 - w_3)$$

$$c_{k+1} = w_1 + d(n-k-1)(w_2 - w_1)$$

and we want to adjust d so that $|c_{k+1} - c_k|^2 = d^2$. This gives a quadratic in d; letting

```
ku23 =    k   (u2-u3)
kv23 =    k   (v2-v3)
ku21 = (n-k-1)(u2-u1)
kv21 = (n-k-1)(v2-v1)
```

it is given by

$$
\begin{aligned}
&d^2\,(-1 - 2ku21\ ku23 - 2kv21\ kv23 + ku21^2 + ku23^2 + kv21^2 + kv23^2) + \\
&d\,(2ku21\ u1 - 2ku21\ u3 - 2ku23\ u1 + 2ku23\ u3 + 2kv21\ v1 - 2kv21\ v3 \\
&\qquad - 2kv23\ v1 + 2kv23\ v3) \\
&- 2u1\ u3 - 2v1\ v3 + u1^2 + u3^2 + v1^2 + v3^2.
\end{aligned}
$$

We want the smaller of the two positive roots.

So far luminance differences between colors have hardly been mentioned. An important detail, at least on our monitor, is that the color vectors must be normalized so that $\max(R,G,B)=1$, not $R+G+B=1$ or $Y=constant$, or else yellow will disappear. I would have preferred to normalize colors to equal perceived brightness. Schrödinger and Helmholtz [17] suggest that rather than holding *Y* fixed, the way to do this is to pick *Y* to minimize the distance from a reference white.

The nonlinear equations solved to generate sample spectra included the dependence on *Y* assuming the $max(R,G,B)=1$ scaling. This gave significantly different colors than when the luminance difference terms were omitted. See Figures 7 and 8.

The weak point in the luminance calculations is the calibration of the monitor. Cowan's instructions [18] were followed but the various techniques for gun normalization gave inconsistent results. Therefore the computed *Y* value is not an accurate luminance value. The resulting slight variations in luminance from color to color have the useful side effect in color level plots of accentuating, due to Mach banding, the boundary between colors. (This may also make the boundaries visible to the color-blind, but no such individuals were available to test the conjecture.) This boundary is a contour line of the function and therefore one the the key features of the plot. Hence, although I would welcome advice on proper luminance calibration, I am content at present to leave well enough alone.

It is still possible, and in practice has sometimes proven useful, to use luminance as an orthogonal variable. That is, we display two functions according to $d=v(f_1)\,c(f_2)$, where *d* is the displayed color, *v* is a scalar between 0 and 1 proportional to f_1, and *c* is a triple corresponding to a maximally bright color with hue depending on f_2. Of course this only works if f_2 is uninteresting when f_1 is small. Another possibility is to use redundant coding in luminance and hue by setting $f_1=f_2$.

4. Empirical results

I have displayed spectra computed by the techniques in the previous section. The methods based on the color literature are clear winners, but even they produce color steps that appear nonuniform. The FMC-1 formula is the winner in my eyes. Considering its relative simplicity, surprisingly good results were also obtained with the CIE *Luv* metric.

It would be difficult to draw firm conclusions even if a panel of observers were used. Adjustment of the contrast on the monitor changes the colors dramatically, yet there is no obviously correct setting. In any case, the distinctions being drawn here are inherently subjective. Local readers may wish to display the spectra for themselves on the monitor in the Unix room; in directory */usr/ehg/color* on the machine *swift* try the command *ldemo*.

Using this command on the Hitachi monitor with the contrast knob set in the vertical position and the brightness adjusted so that the background is just invisible I observed the following. For 20 colors, FMC and Luv clearly give better spacing near the red end. The last two blues of FMC are too close, but Luv has too broad a range of green. Several observers chose FMC as the best compromise. From red to green, FMC and Luv appear the same. For 30 colors, the hexcone and OSA spectra near the red end are improved. The hexcone greens and cyans almost merge. Brightest blue is a bit obtrusive in FMC and hexcone. The blurred cyan in Luv and the blurred deep blue in FMC result in a tossup. For 64 colors, the green blue transition of OSA has shifted significantly left. Tests of the MacLeod-Boynton and Frome formulas gave spectra with too many greens compared to the number of blues.

Since saturated colors can cause a misperception of area, I have also explored using the hexcone model as before, but with less saturated colors. The computations are similar and no new effects were apparent. Notice however that it does not suffice to simply take the generated saturated colors and add white (or equivalently, use the computed hues and change the saturation parameter in the hexcone model), since the geodesics of constant hue are not straight lines in x,y chromaticity space.

The color graphics literature contains a number of related papers reviewing various aspects of colorimetry, but none specifically solve the problem I am interested in. Alvy Ray Smith's widely read paper [19] emphasizes that hue, saturation, and luminance as more natural coordinates for human interaction than R,G,B and explained how to easily reparametrize. Meyer and Greenberg [20] describe various uniform color scales. Farrell [21] has done some related work but ran into ''practical difficulties'' and returned to manual choice of colors. Recent work [22, 23] seems to imply that for the purposes of color contour plots, the key variable is the angle of the color in x,y space about pure blue. That is, colors between green and blue produce contours that are very difficult to distinguish. The attempt to correct for this using the McLeod-Boynton color space was unsuccessful, so it is not clear what the practical implications of that work are.

An interesting report on color selection for cockpit displays in the Boeing 757/767 [24] selected uniformly spaced points in CIE 1960 Luv space. (My observations indicate that the 1976 Luv space might have been a better choice.) Magenta was chosen as one of the colors, but as a result of experiments the color finally chosen was shifted toward the blue end of the line of magentas. Pilots were found to prefer highly saturated colors.

In summary, I would say that the existing color difference formulas give barely adequate results in this application and I would welcome further work along this line by color psychologists.

5. Practical Use

Having settled on the FMC-1 formula, it is possible to tabulate a transformation in hues, as has been done for monochrome compensation tables [25]. This essentially amounts to computing the monotone function representing arclength along the hexcone boundary, weighted by one of the color difference metrics, and doing inverse interpolation at equally spaced points. The distribution is obtained in the limit as the number of nonlinear equations goes to infinity; convergence is rapid enough that solving for 64 colors and using linear interpolation gives satisfactory results. Figure 9 plots the piecewise linear function based on 64 points and the colors obtained by solving the nonlinear equations for 20 points. Since the hexcone model itself is piecewise linear, there is little point in using smoother interpolation. This formulation of color choice as a correction table for the hexcone model is motivated by the desire to simplify conversion of existing graphics programs.

A C routine converting from hue, saturation, and value to red, green, and blue values according to such a precomputed table is available by electronic mail. Type

```
mail mail netlib@research.att.com
send rainbow.c from graphics
```

and the code will arrive by return mail. The particular table in that code is computed for the Hitachi display, but may give adequate results for other displays. Tables for other displays, color difference formulas, or luminance scalings can be easily generated using the techniques described in section 3.

Acknowledgement. I enjoyed helpful discussions with many people, particularly Tom Duff, Fran Frome, Alan Kaplan, John Krauskopf, Rob Pike, Lynn Wilson, and Qasim Zaidi.

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